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Radio-Frequency Size Effect in Potassium: Relation to Cyclotron Resonance, Helicon Waves, and Sound Waves

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Studies are reported of the transmission of 1- to 100-MHz rf energy through a thin potassium slab when a magnetic field is applied either perpendicular or nearly parallel to the surface of the slab. In the perpendicular-field geometry for fields below the helicon Doppler-shifted cyclotron resonance edge, the phase of the transmitted signal increases linearly with applied field as predicted by Gantmakher and Kaner. Near zero applied field, the signal shows the increase in amplitude and extra $\frac{1}{2}\pi$ phase shift predicted by Baraff. When the field is nearly parallel to the slab surface, an attenuation of the transmitted signal is observed at the value of field where the effective phase velocity of the rf current distribution matches the phase velocity of an acoustic wave.

I. INTRODUCTION

An experimental study of the transmission of rf energy through a thin potassium slab in a dc magnetic field is reported. There are two basic mechanisms by which rf fields penetrate into a metal: propagating wave modes and single-particle excitations. This paper presents the results of a study of the relationship of the single-particle excitations (SPE) to the helicon wave mode and cyclotron resonance together with a study of the interaction of the SPE with transverse sound waves.

A propagating wave mode such as the helicon mode exists when the effective dielectric constant of the metal is predominantly real and positive. The rf field distribution produced by a propagating wave is a damped sinusoid with the $1/e$ damping distance often much larger than the electron mean free path l. On the other hand, the SPE carry rf fields into the metal only to a depth comparable to l. Consequently, the SPE modify the rf field distribution appreciably only when anomalous skin effect conditions prevail: that is, when $l \gg \delta$ where δ is the classical skin depth $(2\pi\omega\sigma_0/c^2)^{-1/2}$. If the specimen thickness d satisfies the inequality $\delta \ll d < l$, then the SPE produce the rf size effects discovered by Gantmakher.[1] The field distribution set up by the SPE depends upon the shape of the trajectories executed by the current carriers, and is often highly nonsinusoidal. Gantmakher has described several phenomena that produce sheets of current that are thin compared with the spacing between sheets.[1]

Our experiments were performed on potassium because it represents nature's closest approximation to a free-electron gas. Shoenberg and Stiles[2] have shown that the Fermi surface of potassium is spherical to one part in 1000, and Grimes and Kip[3] have shown that the electron effective mass is isotropic with the value $m^* = (1.21 \pm 0.02)$ m. Consequently, for the purposes of this paper we consider potassium to be a free-electron gas characterized by the parameters: electron density 1.40×10^{22} cm^{-3}, Fermi velocity $v_F = 0.71 \times 10^8$ cm/sec, Fermi radius $k_F = 0.766 \times 10^8$ cm^{-1}, and electron scattering time $\tau = 4 \times 10^{-10}$ sec which is appropriate for our high-purity potassium at 1.2°K.[4]

The organization of the remainder of this paper is as follows: Section II contains a brief review of the theory pertinent to the transmission of rf energy through a conducting slab when a magnetic field is applied perpendicular to the slab. Our experimental results for this geometry are then presented. Section III contains a brief discussion of the transmission phenomena which occur when a magnetic field is nearly parallel to the faces of a slab. This discussion is followed by a report of our observation of the interaction of SPE with a transverse acoustic wave. The Appendix outlines the experimental technique which was employed.

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[1] V. F. Gantmakher, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1967), Vol. 5, Chap. 5, p. 181.

[2] D. Shoenberg and P. J. Stiles, Proc. Roy. Soc. (London) **A281**, 62 (1964).

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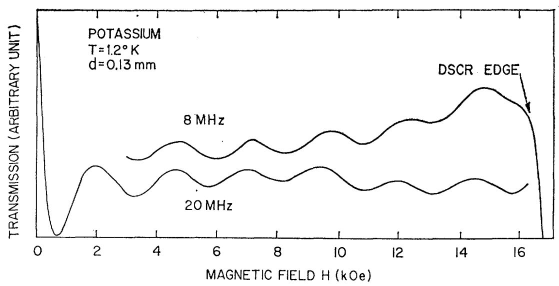

FIG. 1. Output of phase-sensitive detector plotted versus applied field H to display rf signal transmitted through a thin potassium slab in the field normal geometry. The oscillations periodic in H are due to rf currents carried through the specimen by orbiting electrons. The DSCR edge denotes the onset of helicon wave propagation.

II. FIELD NORMAL GEOMETRY

In the field normal geometry the dc magnetic field **H** is applied perpendicular to the faces of a slab-shaped specimen. Applying a field to a free-electron gas causes the current carried into the metal by SPE to precess about **H** at the cyclotron frequency ω_c. Consequently, the current distribution in the electron gas is modified from its zero-field value in a simple way. Chambers[5] and Azbel and Kaganov[6] have shown that the expression for the electric field distribution can be obtained from Reuter and Sondheimer's zero-field expression[7] by making the substitution $\omega\tau\rightarrow\omega\tau\pm\omega_c\tau$ where ω is the frequency of an incident circularly polarized electromagnetic wave. To within an amplitude factor, the electric field distribution in the specimen for a linearly polarized wave becomes

$$E(z)\approx\frac{e^{-z/l}\cos(z/R)}{z^2},$$

where $R=v_F/\omega_c$ is the Larmor radius and the coordinate system is such that the z axis is normal to the slab. This formula is an asymptotic limit taken for $z/l\gg1$. Thus when the mean free path is large $l\gg R$, the electric field oscillates in space with a wave vector given by $k=1/R$.

Gantmakher and Kaner[8] have extended the theory of the penetration of rf fields into metal plates in the field normal geometry. They emphasized that whereas the helicon wave arises from a solution of the equation

$$k^2-4\pi i\omega\sigma_\pm(k)/c^2=0,$$

the SPE arise from the isolated branch points of $\sigma_\pm(k)$. The SPE set up a field distribution in the metal that has a wave vector related to the periodicity of the helical motion of the electrons having the maximum average velocity along **H**. For a general closed Fermi surface the wave vector is given by $k=eHc^{-1}|\partial A/\partial p_z|_{\max}{}^{-1}$, where $A(p_z)$ is the area of the cross section of the Fermi surface at the plane $p_z=\text{const}$. Periodic oscillations in the surface impedance of a Sn plate were observed by Gantmakher and attributed to the SPE. Weisbuch and Libchaber[9] have performed additional studies of the SPE (which they termed Gantmakher waves) in single crystals of copper using a transmission technique.

In a theoretical study of the transmission of electromagnetic energy through a slab, Baraff[10] showed that the SPE lead to a transmission resonance at $\omega=\omega_c$. At cyclotron resonance and with diffuse scattering of electrons at the surface, there is an increase in the amplitude of the transmitted signal and a $\frac{1}{2}\pi$ extra phase shift on each side of $\omega=\omega_c$ (see Baraff's paper for a detailed discussion). The predicted shape of the transmission peak at cyclotron resonance is the square root of a Lorentzian. The π phase shift arises as the current changes from lagging to leading the electric field with increasing H. The calculation is done in the limit $z/l>1$.

We have studied the penetration of rf fields into potassium at frequencies ranging from 1 to 100 MHz using the experimental technique outlined in the Appendix. Representative experimental curves are shown in Fig. 1 for rf transmission through a slab $\approx$0.1 mm thick. When the frequency is 8 MHz the Doppler-shifted cyclotron resonance edge for the helicon wave occurs near 16 kOe. As the field is increased above 16 kOe the amplitude of the transmitted helicon wave rapidly increases. In Fig. 1, the transmitted signal decreases and becomes negative above 16 kOe because the phase of the helicon wave is π relative to the reference signal of the phase-sensitive detector. The various phenomena that occur in the helicon wave regime were presented in an earlier paper.[11]

Below 16 kOe the SPE produce an oscillatory variation in the transmitted signal with a period independent of frequency (periodicity in H the same at 8 and 20 MHz). The wave vector of the SPE, $k=1/R$, increases linearly with H. In contrast, the wave vector of the helicon wave equals $1/R$ at the Doppler-shifted cyclotron resonance edge and then decreases with increasing H. Thus at fields such that $H<H_{\text{edge}}$ the current distribution in the metal has the wave vector of the SPE. When $H=H_{\text{edge}}$ the SPE and the helicon wave have the same wave vector, and for $H>H_{\text{edge}}$ the current distribution has the wave vector of the helicon wave.

To analyze the data we note that the phase of the transmitted signal changes by 2π between any two successive transmission maxima. The phase φ and

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[6] M. Ya. Azbel and M. I. Kaganov, Dokl. Akad. Nauk SSSR, **95**, 41 (1954).
[7] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) **A195**, 336 (1948).
[8] V. F. Gantmakher and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. **48**, 1572 (1965) [English transl.: Soviet Phys.—JETP **21**, 1053 (1965)].
[9] G. Weisbuch and A. Libchaber, Phys. Rev. Letters **19**, 498 (1967).
[10] G. A. Baraff, Phys. Rev. **167**, 625 (1968).
[11] A. Libchaber and C. C. Grimes, Phys. Rev. **178**, 1145 (1969).

amplitude of the 20-MHz signal are plotted against H in Fig. 2. The reciprocal of the slope of the phase plot yields the periodicity $\Delta H = 2.47 \pm 0.1$ kOe. For a general closed Fermi surface the predicted periodicity is[8]

$$\Delta H = \frac{c}{ed} \left| \frac{\partial A}{\partial p_z} \right|_{\max}, \tag{1}$$

which for a free-electron gas becomes

$$\Delta H = (2\pi c/ed) p_F. \tag{2}$$

Using the free-electron value of the Fermi momentum $p_F = 7.86 \times 10^{-20}$ g cm/sec and the specimen thickness d in (2) the predicted periodicity is 2.32 ± 0.12 kOe which agrees with the observed value within the experimental error. The largest uncertainty in the experiment is in the determination of the specimen thickness. The potassium specimens are so soft that it is difficult to measure the thickness directly with a micrometer or Vernier caliper. Consequently, we have utilized the rf size effect in the field parallel geometry to determine the thickness. Our data yield the value $d = 0.133 \pm 0.007$ mm. Since we have used the free-electron value of the Fermi momentum in the determination of the specimen thickness, the agreement between predicted and observed periodicities only shows that the free-electron model is consistent with all the data, i.e., the Fermi momentum is the same when measured along **H** or perpendicular to **H**.

Overhauser has suggested that a charge density wave ground state may be appropriate for potassium.[12] The model that he has advanced leads to a "lemon-shaped" Fermi surface with the axis of symmetry parallel to the magnetic field. Using Eq. (1) the periodicity for the SPE deduced from Overhauser's model is 17% less than the free-electron value. Our data are not consistent with the Overhauser model.

Note that in Fig. 1 both phase and amplitude information are present. To obtain the experimental curves, the phase of the lock-in detector reference was adjusted to maximize the signal at $H = 0$. The phase φ, plotted in Fig. 2, is measured relative to the phase of the transmitted signal at $H = 0$. From Fig. 2 it is clear that φ increases linearly with H for fields above 1 kOe. The extra $\frac{1}{2}\pi$ phase shift that occurs near $H = 0$ is just the phase shift that Baraff predicted would occur at cyclotron resonance. (At 20 MHz the cyclotron resonance field is only 8 Oe.) The extra $\frac{1}{2}\pi$ phase shift occurs in the first SPE half-period because the cyclotron resonance linewidth $\delta H_{CR} = m^* c/e\tau \approx 200$ Oe is less than the SPE half-period $\delta H \approx 1200$ Oe.

Assuming diffuse surface scattering, Baraff[10] predicted that the amplitude of the SPE signal should vary as the square root of a Lorentzian near cyclotron resonance. Gantmakher and Kaner[8] assumed specular surface scattering and predicted that the signal amplitude should be independent of field far from resonance. Both

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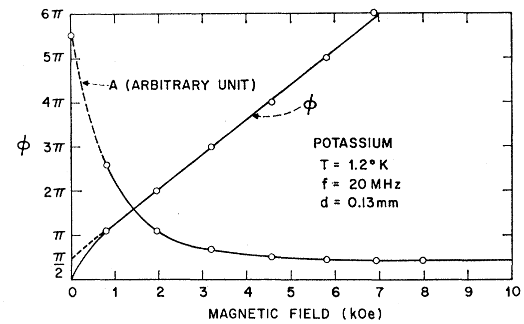

FIG. 2. Plots of phase φ and amplitude A of rf signal transmitted through a potassium slab. Data points were taken from the 20-MHz curve in Fig. 1.

calculations were in the asymptotic limit $z/l \gg 1$. Our experimental conditions are such that $z/l \sim \frac{1}{3}$ which is out of the range of the calculations. Nevertheless, our amplitude data points in Fig. 2 appear to display both of the predicted effects.

The physical picture near cyclotron resonance is interesting. When a circularly polarized wave is incident on a metallic slab, orbiting electrons (SPE) carry currents into the bulk at velocity v_F and rotate the direction of the current at angular frequency ω_c. Consequently, the current distribution in the bulk has a spiral-wave-like character with an effective phase velocity that is field-dependent $v_{\text{phase}} = \omega v_F/(\omega \pm \omega_c)$. The effective phase velocity goes to infinity and changes sign as the cyclotron frequency is swept through resonance. Thus the spiral current distribution has a field-dependent pitch—the sense of rotation is different on either side of cyclotron resonance and the pitch goes to infinity at resonance. (Baraff[10] gives a more detailed description of effects near cyclotron resonance including a description of the transmitted amplitude in terms of Fresnel zones on the Fermi surface.)

III. FIELD PARALLEL GEOMETRY

The field parallel geometry gives rise to the rf size effect which has been exploited by Gantmakher and others in numerous Fermi-surface studies.[1] Since detailed studies of the rf size effect in potassium have been presented by Koch and Wagner,[13] Peercy *et al.*,[14] and Tsoy and Gantmakher,[15] our work in the field parallel geometry was confined to the interaction between SPE and acoustic waves.

When **H** is accurately parallel to the specimen surface and the incident rf wave is polarized with its electric field perpendicular to **H**, sharp anomalies in the rf transmission appear at those values of H where an integral number of extremal dimension orbits just span the sample thickness, i.e., those values of H for which

[13] J. F. Koch and T. K. Wagner, Phys. Rev. **151**, 467 (1966).
[14] P. S. Peercy, W. M. Walsh, Jr., L. W. Rupp, Jr., and P. H. Schmidt, Phys. Rev. **171**, 713 (1968).
[15] V. S. Tsoy and V. F. Gantmakher, Zh. Eksperim. i Teor. Fiz. **56**, 1232 (1969).

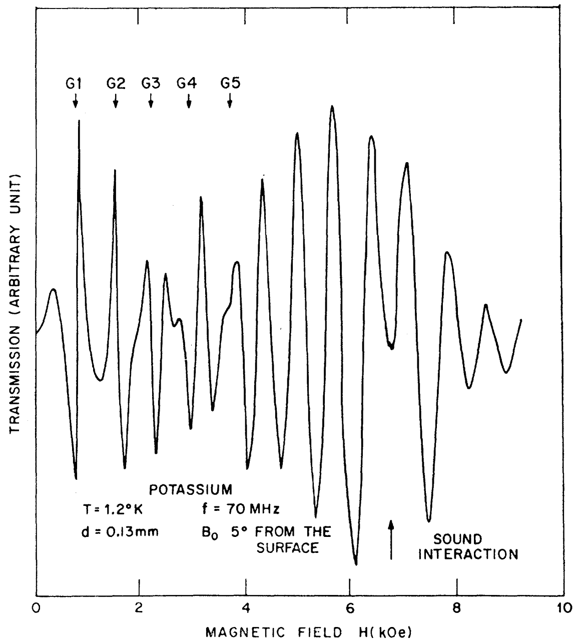

FIG. 3. Output of phase-sensitive detector plotted against applied field to show rf signal transmitted through a thin potassium slab when the field is inclined 5° relative to the surface of the slab. An arrow at 6.7 kOe denotes the point where the signal is attenuated due to phase matching of the rf current distribution and an acoustic wave.

$n(2R)=d$, where n is an integer. The transmission anomalies are due to thin sheets of current which are set up in the bulk of the metal by the orbital motion of the electrons and require $\omega_c\tau \gtrsim 1$ for their existence. The amplitude of the nth current sheet is proportional to $(\delta/2R)^{n/2}$ so successive sheets are rapidly damped.[1] Such rapid damping reflects the fact that only a small fraction of the electrons $\sim\delta/2R$ contribute to the current in a given sheet; i.e., the effective zone on the Fermi surface is very small.

A nearly sinusoidal lightly damped current distribution is set up in the specimen if **H** is inclined a few degrees relative to the sample surface.[8,13–15] In an inclined field the effective zone on the Fermi surface becomes much wider and the fraction of the electrons contributing to the current increases; consequently, the current distribution is damped much more slowly than in the field parallel geometry. Figure 3 shows an experimental trace of a 70-MHz transmitted signal as a function of H for an inclination of 5°. The first five current sheets give rise to the relatively narrow transmission anomalies labeled G_1 through G_5. At higher fields the anomalies merge into a nearly sinusoidal variation in the transmission which corresponds to a nearly sinusoidal current distribution in the metal near the receiver side of the specimen. Although the current distribution is nearly sinusoidal it does not represent a true wave obtained from a dispersion relation, but rather it is simply the result of the orbital motion of the current carrying electrons or SPE.[14] Still, we can ascribe to the current distribution an effective phase velocity ω/k, where k is the dominant Fourier component of the current distribution given by $k\approx\pi/R$. When the effective phase velocity of the current distribution equals the phase velocity of a transverse acoustic wave, a strong interaction occurs between the two excitations.[16] No detailed calculations on this particular interaction have been performed. However, Skobov and Kaner[17] have calculated the interaction of acoustic waves with several different electromagnetic wave modes. One of the modes they treated appears to be very similar to the wavelike excitation we are considering.[18] Their "waves with a discrete spectrum" were shown to couple with acoustic waves through both the electromagnetic fields accompanying the waves and through the deformation potential.

The acoustic wave interaction appears in Fig. 3 at 6.7 kOe. Note that k depends only on H and not on ω. Consequently, the current distribution in the metal and the rf transmission are expected to be essentially independent of frequency except for the anomaly that occurs at the acoustic-wave interaction. The field where the acoustic-wave interaction occurs is expected to be proportional to ω. We have observed just such behavior. Experimental traces at frequencies from 40 to 72 MHz are very similar except for the anomaly which we ascribe to the acoustic-wave interaction. The field where this anomaly appears is proportional to the signal frequency. From our data we deduce an effective phase velocity of the current distribution of 1.0×10^5 cm/sec. This phase velocity is consistent with the measurements of Marquardt and Trivisonno[19] who found transverse acoustic velocities in potassium at 4.2°K ranging from 0.646 to 1.78×10^5 cm/sec depending on propagation direction and polarization.

ACKNOWLEDGMENTS

It is a pleasure to thank P. H. Schmidt who purified the potassium used in the experiments. We have benefited from numerous discussions with our colleagues

[16] Coupling between SPE and acoustic waves should also occur in the field normal geometry. We were unable to observe such coupling because the experimentally accessible frequencies were so low that the two excitations could only interact over a distance of about two wavelengths.

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[18] V. G. Skobov and E. A. Kaner predicted that "waves with a discrete spectrum" should occur for values of ω, H, $\omega_c\tau$ and field inclination angle similar to the values employed in our experiments and the experiments of P. S. Peercy *et al.* (Ref. 14) and P. S. Peercy and W. M. Walsh, Jr. [Phys. Rev. Letters **17**, 741 (1966)]. However, for fixed values of H and inclination angle the waves are predicted to occur at discrete values of ω; whereas experimentally wavelike excitations are observed at any frequency from a few MHz to 72 MHz, the highest frequency studied. The wave vector of the observed excitation ($k\approx\pi/R$) is very close to the smallest value ($k_1=5\pi/4R$) of the wave vector predicted for the "waves with a discrete spectrum."

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G. A. Baraff, P. S. Peercy, P. M. Platzman, and W. M. Walsh, Jr. We also want to thank V. G. Skobov for an informative discussion.

APPENDIX: EXPERIMENTAL METHODS

The experimental configuration which we have employed to transmit rf energy through thin slab-shaped specimens (≈ 0.1 mm thick $\times 10$ mm diam) is similar to that of Grimes and Buchsbaum.[20] In this configuration, a receiver coil and a transmitter coil are placed near the two faces of the specimen. The coils consist of a few turns (2 turns at the highest frequencies) of No. 40 AWG copper wire wound on a Teflon form which has a rectangular cross section. Each coil is embedded in Epoxy in a square opening in one face of a brass housing. The face of each housing is lapped flat and the turns of the coil are recessed back from the face by only ≈ 0.02 mm. When a specimen is mounted between the two coils and rf current is passed through the transmitter coil, eddy currents are induced at the surface of the specimen due to the near-field pattern of the coil. The launching wave is thus a linearly polarized plane wave that propagates perpendicular to the surface of the specimen. At the opposite face a part of the transmitted energy is coupled to the receiver coil and fed to a continuously tunable phase-sensitive detector[21] which derives its phase reference from the transmitter. The output of the phase-sensitive detector is displayed on an X-Y recorder as a function of the applied magnetic field. The displayed signal is then an interference pattern (Fig. 1) as the transmitted signal becomes alternately in and out of phase with the reference signal.

To obtain a satisfactory signal-to-noise ratio when looking at small signals as in Fig. 1, it is necessary to achieve good isolation between transmitter and receiver circuits. In earlier work, the potassium specimens were completely covered with Mylar sheets to retard oxidation of the potassium. Such specimens were thus insulated from the brass coil housings and the isolation between transmitter and receiver coils was poor. To obtain better isolation we have devised a method of specimen preparation that allows the specimen to be electrically grounded to the coil housings. With this technique -120-dB isolation has been achieved.

To permit electrical grounding of the highly reactive potassium specimen, only the center portion of the specimen is covered with Mylar while the outer portion is covered with a washer-shaped gold foil which is in good electrical contact with the potassium. The specimen is prepared in an argon atmosphere by pressing to the desired thickness a stack of materials consisting of, first, a clean annealed gold-foil washer 0.03 mm thick, second, a disk of Mylar 0.006 mm thick and slightly larger in diameter than the opening in the gold washer, third, a lump of freshly cut potassium which is free of oxide and protective coatings, fourth, a ring-shaped shim which surrounds the potassium and serves to determine the approximate thickness of the whole assembly after pressing, fifth, another Mylar disk, and sixth, another gold washer. When the assembly is pressed between flat plates the potassium flows radially outward and makes good metal-to-metal contact with the gold washers. The portions of the specimen which are positioned opposite the transmitter and receiver coils are visible through the Mylar windows. These surfaces are flat and have a bright metallic luster. Although the completed specimens can be stored in the atmosphere at room temperature for several hours before corroding appreciably, they are normally stored in liquid nitrogen until they are mounted in the apparatus. The specimen is mounted between the faces of the coil housings under moderate spring pressure. Good electrical contact between the potassium and the gold washers and between the washers and the faces of the coil housings produces the desired isolation between coils. The resulting specimens are polycrystalline, but with large crystallites (typical dimensions of 1 mm or more). It is not unusual for a single crystallite to occupy most of the window area of the specimen.

[20] C. C. Grimes and S. J. Buchsbaum, Phys. Rev. Letters **12**, 357 (1964).

[21] J. W. Hansen, C. C. Grimes, and A. Libchaber, Rev. Sci. Instr. **38**, 895 (1967).